Corporate Finance and Incentives Winter Exam December 21 2009 – Solutions Manual.

Problem 1: Factor Models

1) Find the absolute portfolio weights to create each of the three possible pure factor portfolios.

To find the Pure Factor Portfolio weights we invert the asset beta matrix
Use stocks along rows and risk factors along columns

$$\beta_{Assets} X^T = I(3) \Leftrightarrow$$
$$\beta_{Assets}^{-1} \beta_{Assets} X^T = \beta_{Assets}^{-1} I(3) \Leftrightarrow$$
$$X^T = \beta_{Assets}^{-1} \Leftrightarrow$$
$$X = \left(\beta_{Assets}^{-1}\right)^T$$

b matrix (3 x 3)

Factor betas

		Factor		
		Unemployment	Cons. Confid.	Producer Prices
Asset	Gizmo Corp.	1,20	0,80	1,10
	ACME Bricks	0,80	1,10	1,20
	Widget Ltd.	0,90	0,80	0,70

Pure factor portfolio weights (SOLUTION)

		Pure factor pf		
		Unemployment	Cons. Confid.	Producer Prices
Asset	Gizmo Corp.	**0,96**	**-2,64**	**1,78**
	ACME Bricks	**-1,62**	**0,76**	**1,22**
	Widget Ltd.	**1,27**	**2,84**	**-3,45**

Find portfolio weights
From formulas. I = b * X^T.
Isolate X^T = (b)^-1

Take transpose of X to display Stocks as rows and risk factors as columns

Pure factor beta contributions (Result check)

		Pure factor pf		
		Unemployment	Cons. Confid.	Producer Prices
Asset	Gizmo Corp.	1,16	-2,11	1,95
	ACME Bricks	-1,30	0,84	1,46
	Widget Ltd.	1,14	2,27	-2,42
	Sum contrib.	**1,00**	**1,00**	**1,00**

Each asset's factor contribution = asset weight (x) * asset's factor beta (b)

Pure factor portfolio betas (Result check)

		Pure factor pf		
		Unemployment	Cons. Confid.	Producer Prices
Pure factor portfolio	PFP Unempl.	1	0	0
	PFP Cons Conf	0	1	0
	PFP PPI	0	0	1

Simply sumproduct of factor betas * pure factor portfolio weights = b * X^T = I(3)

2) What are the three risk factor betas for the pure factor portfolio that only has risk exposure to Consumer Confidence?

The Consumer Confidence pure factor portfolio has by definition:

$$\beta_{PFP(Consumer-Conf),Unemployment} = 0$$
$$\beta_{PFP(Consumer-Conf),ConsumerConfidence} = 1$$
$$\beta_{PFP(Consumer-Conf),PPI} = 0$$

3) What is the expected return on the pure factor portfolio that only has risk exposure to Consumer Confidence given the observed forecast?

The pure factor portfolio for consumer confidence has the following form:

$$R_{PFP(Consumer-Conf)} = \alpha_{PFP(Consumer-Conf)} + 0 \cdot Unemployment + 1 \cdot ConsumerConfidence + 0 \cdot PPI$$

$$R_{PFP(Consumer-Conf)} = 4\% + 1 \cdot 2\% = 6\%$$

4) What return would the factor model predict for Compost Esq.?

$$r_{Compost\ Esq.} = 8\% + 0.90 \times Unemployment + 1.20 \times Consumer\ Confidence + 1.00 \times PPI$$

$$r_{CompostEsq} = 8\% + 0.9 \cdot 2\% + 1.2 \cdot 4\% + 1.0 \cdot (-3\%) = 11.6\%$$

5) Create the tracking portfolio that mimics the risk behaviour of Compost Esq. by utilising the three different pure factor portfolios found in 1). What are the asset weights for Gizmo Corp., ACME Bricks, Widget Ltd. and the risk-free rate?

A2: A tracking portfolio with the same sensitivities to the risk factors as asset *i* would simply be a weighted combination of a risk-free asset and the pure factor portfolios for each of asset *i*'s risk factors with weights:

$$\beta_{i1} \text{ for pure factor portfolio 1}$$
$$\beta_{i2} \text{ for pure factor portfolio 2}$$
$$\vdots$$
$$\beta_{ik} \text{ for pure factor portfolio k}$$
$$1-\sum_{j=1}^{k}\beta_{ij} \text{ for the risk free asset}$$

(Result 6.5, p. 198)

So we have a Tracking Portfolio Return model as follows:

$$R_{TrackingPortfolio} = (1-\sum_{j=1}^{k}\beta_{CompostEsq,j})\cdot r_f + \beta_{CompostEsq,Unemployment}\cdot(x_{Gizmo,PFP(Unemployment)} + x_{ACME,PFP(Unemployment)} + x_{Widget,PFP(Unemployment)})\cdot Unemployment$$
$$+\beta_{CompostEsq,ConsumerConfidence}\cdot(x_{Gizmo,PFP(ConsumerConfidence)} + x_{ACME,PFP(ConsumerConfidence)} + x_{Widget,PFP(ConsumerConfidence)})\cdot ConsumerConfidence$$
$$+\beta_{CompostEsq,PPI}\cdot(x_{Gizmo,PFP(PPI)} + x_{ACME,PFP(PPI)} + x_{Widget,PFP(PPI)})\cdot PPI$$

The Pure Factor Portfolios have the following functional forms:

PFP(Unemployment) =	alpha(PFP(Unemployment)) + 0.9 x (0.96 x Gizmo - 1.62 x ACME + 1.27 x Widget)
PFP(Cons Conf) =	alpha(PFP(Cons Conf)) + 1.2 x (-2.64 x Gizmo + 0.76 x ACME + 2.84 x Widget)
PFP(PPI) =	alpha(PFP(PPI)) + 1.0 x (1.78 x Gizmo + 1.22 x ACME - 3.45 x Widget)

Tracking Portfolio asset weights are thus:

$$x_{Gizmo,TrackingPortfolio} = \beta_{CompostEsq,Unemployment}\cdot x_{Gizmo,PFP(Unemployment)} + \beta_{CompostEsq,ConsumerConfidence}\cdot x_{Gizmo,PFP(ConsumerConfidence)} + \beta_{CompostEsq,PPI}\cdot x_{Gizmo,PFP(PPI)} = 0.9\cdot 0.96 + 1.2\cdot(2.64) + 1.0\cdot 1.78 = 0.52$$

$$x_{ACME,TrackingPortfolio} = \beta_{CompostEsq,Unemployment}\cdot x_{ACME,PFP(Unemployment)} + \beta_{CompostEsq,ConsumerConfidence}\cdot x_{ACME,PFP(ConsumerConfidence)} + \beta_{CompostEsq,PPI}\cdot x_{ACME,PFP(PPI)} = 0.9\cdot(-1.62) + 1.2\cdot(0.76) + 1.0\cdot 1.22 = 0.67$$

$$x_{Widget,TrackingPortfolio} = \beta_{CompostEsq,Unemployment}\cdot x_{Widget,PFP(Unemployment)} + \beta_{CompostEsq,ConsumerConfidence}\cdot x_{Widget,PFP(ConsumerConfidence)} + \beta_{CompostEsq,PPI}\cdot x_{Widget,PFP(PPI)} = 0.9\cdot(1.27) + 1.2\cdot 2.84 + 1.0\cdot(-3.45) = 1.10$$

$$x_{Riskfree} = 1 - \sum_{j=1}^{k}\beta_{CompostEsq,j} = 1 - (0.9 + 1.2 + 1) = -2.10$$

Result check for Tracking Portfolio (Portfolio betas = Compost Esq. beta)

Beta Unemployment = x(Gizmo) * beta(Unemployment,Gizmo)+x(ACME) * beta(Unemployment,ACME)+x(Widget) * beta(Unemployment,Widget)
= -0.52 * 1.2 + 0.67 * 0.8 + 1.10 * 0.9 = **0,90**
Beta Consumer Confidence = x(Gizmo) * beta(Consumer Confidence,Gizmo)+x(ACME) * beta(Consumer Confidence,ACME)+x(Widget) * beta(Consumer Confidence,Widget)
= - 0.52 * 0.8 + 0.67 * 1.1 + 1.1 * 0.8 = **1,20**
Beta PPI = x(Gizmo) * beta(PPI,Gizmo)+x(ACME) * beta(PPI,ACME)+x(Widget) * beta(PPI,Widget)
= - 0.52 * 1.1 + 0.67 * 1.2 + 1.1 * 0.7 = **1,00**

6) Is there an arbitrage opportunity? If so, how would you create arbitrage profits and would the arbitrage returns be?

The expected tracking portfolio return is 10%
The expected Compost Esq. return is 8%

Since these two investments have identical systematic risks but different returns there is an arbitrage opportunity.

An investor could simply borrow at 8% by taking a short position in Compost Esq. and invest the proceeds in the tracking portfolio for an expected return of 10%.

This would yield a net return of 10% - 8% = 2% on the investment at zero risk and with no initial capital needed

Problem 2: Real Investments

1) What is the net present value (NPV) to the firm of investing in project A and B respectively?

Risk free	1,5%		38	Fixed cost

Remember that the value of each project must be evaluated by its <u>incremental</u> Cash flows
That is cash flows from including a project relative to those of not including the project.

	Sub. Cost		12	26
	t	A	B	
Ini. Outlay		0	-75	-50
Cost saving		1	26	12
		2	26	12
		3	26	12
		4	26	12
		5		12
		6		12
		7		12
		8		12
	NPV		25,21	39,83

2) What two methods may be used by an investor with long term horizon to compare the attractiveness of two mutually exclusive types of investment projects? Describe each method briefly.

Two methods to comparing mutually exclusive projects with different horizons:
a) Least common multiple of lives method (LCML)
b) Equivalent annual annuity method (EAA)

LCML method assumes that some or all projects can be replicated such that each project or string of replicated identical projects has an investment horizon equal to the least common multiple of individual project lives.

As an example the least common multiple of lives between project A and B would be 8 years assuming that project A could be replicated at the end of the 4th year.

The project or string of replicated identical projects with an investment horizon equal to the LCML that has the highest NPV identifies the profit maximizing investment opportunity for a long term investor.

EAA method calculates the annuity (incremental) cash flow that corresponds to a project with a principal value equal a project's NPV over its lifetime at an interest rate equal to the discount rate. The project that is found to have the highest annuity (incremental) cash flow is the most attractive to a long term investor.

3) Assume the firm has an 8-year investment horizon and that project A can be replicated immediately at the end of the 4th year. How should a profit maximizing firm invest if it has no budget restrictions? Calculate the corresponding net present value (NPV).

New incremental cash flow profile of projects:

	t	A x 2	B
Ini. Outlay	0	-75	-50
Cost saving	1	26	12
	2	26	12
	3	26	12
	4	-49	12
	5	26	12
	6	26	12
	7	26	12
	8	26	12
NPV		48,97	39,83

(Old annual cost - New annual cost) = (38 - 26) (→ year 1 cost saving)

(Old annual cost - New annual cost) - Initial Investment outlay= (38 - 12) - 75 (→ year 4: -49)

A profit maximizing investor with an 8 year investment horizon would optimally invest in project A and replicate the same project at the end of the 4th year to an increase in the firm's net present value of 48,97

4) Calculate the present value (PV) and the net present value (NPV) of each project.

Profitability Index (PI) = PV/-Cost <=> PV = PI x (-Cost)
NPV = PV - Cost

Project	Cost	PI		PV	NPV
C	960	4,17		4003	3.043
D	360	3,34		1202	842
E	800	4,80	=>	3840	3.040
F	120	2,68		322	202
G	640	3,76		2406	1.766

In decreasing order the most profitable projects are:

Project	Cost	PV	PI	NPV
E	800	3840	4,80	3.040
C	960	4003	4,17	3.043
G	640	2406	3,76	1.766
D	360	1202	3,34	842
F	120	322	2,68	202

5) Assume you have a budget constraint of 1700. What investments would maximize the investors profit (NPV) given his budget constraint? What is the NPV?

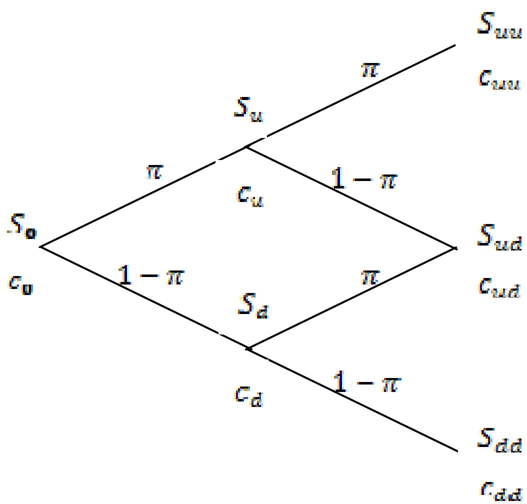
Constraint	1700	
Projects E + G + F		cost 1560 and have a 5008 NPV

6) Why would you not necessarily choose the investments with the top Profitability Index scores?

An investor would not necessarily always choose projects with the highest PI if they have a budget constraint. This is because a budget constraint may sometimes prevent him/her from choosing freely between projects. For each project the investor accepts to invest in, his set of feasible additional investments is reduced due to a lowered amount of free capital. A profit maximizing investor with a given budget constraint should seek to maximize the sum of his projects' total present value while keeping his expenses within his budget.

Problem 3: Options

1) Set up the binomial tree.



$$u = e^{\sigma\sqrt{(T/N)}} = e^{0.22314\times\sqrt{1}} = 1,25$$

$$d = \frac{1}{u} = 0,8$$

$$\pi = \frac{1 + r_f - d}{u - d} = \frac{2}{3}$$

$$S_u = S_0 u = 125$$

$$S_d = S_0 d = 80$$

$$S_{ud} = S_0 ud = 100$$

$$S_{uu} = S_0 uu = 156,25$$

$$S_{ud} = S_0 dd = 64$$

2) Estimate the price of the European call option.

$$c_0 = \frac{(\pi V_u + (1-\pi)V_d)}{(1+r_f)^1} = 16,99$$

$$c_u = \frac{(\pi V_{uu} + (1-\pi)V_{ud})}{(1+r_f)^1} = 28,03$$

$$c_d = \frac{(\pi V_{ud} + (1-\pi)V_{dd})}{(1+r_f)^1} 0$$

$$c_{uu} = max(S_{uu} - K, 0) = 46,25$$

$$c_{ud} = max(S_{ud} - K, 0) = 0$$

$$c_{dd} = max(S_{dd} - K, 0) = 0$$

3) Briefly comment on what the price of a comparable American call option would be.

The price will be identical as an American call will never be exercised early when the underlying assets is non-dividend paying. Hence the price must equal the price of a European call. The American call will not be exercised early for two reasons. First, due to time value of money it is better the later the strike price is paid. Second, the call option provides an insurance if price of the underlying asset decreases. Thus, you would rather sell the call option prior to maturity than exercise it.

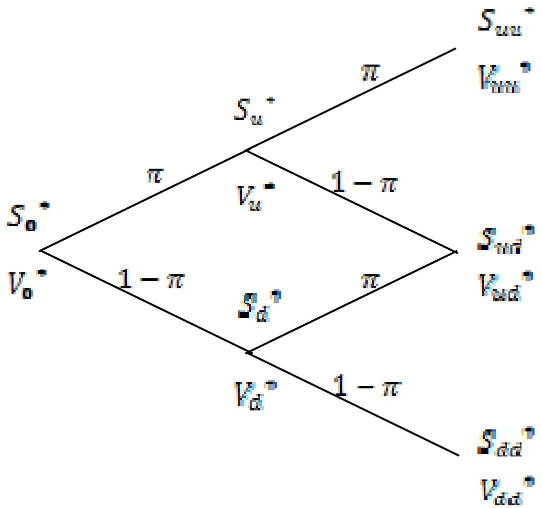
4) Estimate the price of the European and American call option when Hijacking Inc. it pays a dividend and briefly interpret the result. (hint: strip the share price of dividends)

Stripped dividends:

$$PV(Div_0) = \frac{20}{(1+0,1)^{1,5-0}} = 17,34$$

$$PV(Div_1) = \frac{20}{(1+0,1)^{1,5-1}} = 19,07$$

New European binomial tree:



$u = e^{\sigma\sqrt{(T/N)}} = e^{0.22314x\sqrt{1}} = 1,25$

$d = \frac{1}{u} = 0,8$

$\pi = \frac{1 + r_f - d}{u - d} = \frac{2}{3}$

$S_0^* = S_0 - PV(Div_0) = 82,66$

$S_u^* = S_0^* u = 103,32$

$S_d^* = S_0^* d = 66,13$

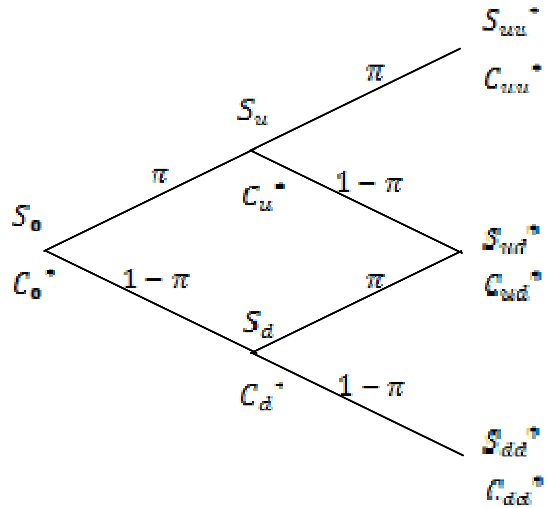
$S_{ud}^* = S_0^* ud = 82,66$

$S_{uu}^* = S_0^* uu = 129,16$

$S_{ud}^* = S_0^* dd = 52,91$

$c_0^* = 7,04$

American binomial tree



$u = e^{\sigma\sqrt{(T/N)}} = e^{0.22314x\sqrt{1}} = 1,25$

$d = \frac{1}{u} = 0,8$

$\pi = \frac{1 + r_f - d}{u - d} = \frac{2}{3}$

$S_0^* = S_0 - PV(Div_0) = 82,66$

$S_u = S_u^* + PV(Div_1) = S_0^* u + PV(Div_1) = 122,40$

$S_d = S_d^* + PV(Div_1) = S_0^* d + PV(Div_1) = 85,20$

$S_{ud}^* = S_0^* ud = 82,66$

$S_{uu}^* = S_0^* uu = 129,16$

$S_{ud}^* = S_0^* dd = 52,91$

$C_0^* = 7,51$

$C_0^* = max\left(\frac{(\pi V_u + (1-\pi)V_d)}{(1+r_f)^1}, S_0 - K\right) = 7,51$

$C_u^* = \max\left(\frac{(\pi V_{uu} + (1-\pi)V_{ud})}{(1+r_f)^1}, S_u - K\right) = 12,40$

$C_d^* = max\left(\frac{(\pi V_{ud} + (1-\pi)V_{dd})}{(1+r_f)^1}, S_d - K\right) = 0$

$c_{uu} = max(S_{uu} - K, 0) = 19,16$

$c_{ud} = max(S_{ud} - K, 0) = 0$

$c_{dd} = max(S_{dd} - K, 0) = 0$

The American call is worth more than the European call as it will be optimal to exercise the American option prematurely.

5) Briefly interpret the difference in the price of the American call option when Hijacking Inc. pays and does not pay a dividend.

The price of the American call is now lower. The American call is exercised early due to the dividend payment which hollows the value of the underlying asset subsequent to the dividend payment. Thus, as an option holder you are not (necessarily) compensated for this. Hence you may want to exercise an American call just before the dividend payment such that you receive the dividend. Notice, that when dividends were not paid the holder of the call option would still get the dividend at a later date. Thus, the dividend forces the holder to exercise early. Furthermore, the share price development is less volatile when the share is stripped of dividend. This also implies a lower value of the call option as the value increases with volatility.

6) Briefly comment on whether it is possible to estimate the price of a comparable American put using the put-call parity.

It is not as the put-call parity does not hold strictly for American options. However, it possible to estimate some boundaries for the price of the American put using the put-call parity for American options:

$$S_0 - K \le C_0 - P_0 \le S_0 - PV(K)$$

Problem 4: Essay Questions

1) Define the concepts duration and convexity and explain what they can be used for.

Duration and convexity are measures for how sensitive the price of a bond is to interest rate changes.

State the definitions

$$D = \frac{1}{PV(C,y)} \sum_{t=1}^{T} t \frac{c_t}{(1+y)^t}$$

$$D = \sum_{t=1}^{T} t w_t, \text{ where } \quad w_t = \frac{c_t}{(1+y)^t} / \frac{1}{PV(C,y)}$$

$$K = \sum_{t=1}^{T} t^2 w_t$$

Interpretation – (Macaulay) Duration:
1: Duration (D) is a bond’s interest rate risk. A 1% interest rate increase/decrease causes an approximately D% price decrease/increase.
2: Duration can also be interpreted as the length of time that a bond can ensure an average annual return equal to the bond’s rate r.

Interpretation – (Macaulay) Convexity:
Convexity is a measure of the curvature of the price as a function of the interest rate, i.e., how much the duration changes when the interest rate changes.

It is based on a flat term structure such that interest changes can only be horizontal shifts in this.

Duration and convexity can be used to make “back of the envelope” calculations about how bond prices would react to changes in interest rates. For instance, a high duration bond would fall more in value if interest rates go up than a low duration bond.

Additionally, duration and convexity can be used for immunization: making bond portfolios immune to interest rate changes.

(Modified duration and convexity shows the relative change w.r.t. to a change in interest rate)

2) Explain why comparable firms preferably should be used when trying to estimate the price of a firm based on multiples.

Consider and/or P/E and EV/EBITDA (EV/EBIT). (P denotes the market price of equity)

Assume for simplicity that a firm can be valued as:

$$EV = \frac{FCF}{wacc - g}$$

$P = EV - D_t$ $D =$ Market value of debt

- P and EV depend on the WACC which depends on the risk and capital structure of the firm. Thus, firms with comparable capital structures should be used for comparison.
- The risk of firms is likely most similar for firms of approximately the same size and operating in the same industry
- EBITDA, EBIT, and Earnings are most likely to a higher degree affected by the same non-operating items which may disturb the multiple if they are not corrected for this.
- The growth opportunities are most likely similar for firms of the same size and in the same industry.
    - It is easier to grow for a small firm than a large firm and the growth opportunities for a firm does also depend on the growth opportunities in the industry

3) Discuss which tax issues influences the decision to distribute earnings to shareholders as either dividends or share repurchases.

The amount taxable – favours share repurchase:
- Dividends: Full amount taxed
- Share repurchase: Only the share price increase from the share repurchase which exceeds the share price at purchase is taxed. Hence, the taxable amount of the is between zero and the full amount distributed

Timing of taxation – favours share repurchase:
- Dividends: When received
- Share repurchase: When the share is sold

Tax rates:
- Depends on whether dividends or share repurchases has the highest tax rates. Favours the pay-out policy which has the lowest tax rate

Dividend clientele:
- Trust
    - (May) prefer dividends if they are only allowed to spend income but not capital gains
- Corporations
    - May prefer dividends if they are not fully taxed while capital gains are
- Tax brackets
    - High tax brackets investors prefer share repurchase if dividends are taxed as ordinary income while capital gains are taxed at a fixed rate
- Tax exempt investors

Should in theory be indifferent but they prefer dividends as transaction costs are lower than for share repurchases